first order perturbation method[5] as

$$k_G = k_G^{(\epsilon)}\left\{1 + \frac{1}{4}\frac{s_{11}^2}{(s_{11}^2-1)J_1^2(s_{11})} (\epsilon_F - \epsilon)\left(\frac{k}{k_G^{(\epsilon)}}\right)^2\left(\frac{\rho}{a}\right)^2\right\}. \quad (23)$$

Specializing (20) and (23) to $\epsilon = 1$ and combining them with (17) we obtain the same result as Suhl and Walker[3] [(11) p. 1142] who treated the problem of a thin ferrite rod in an air-filled guide in a more general way. Clarricoats[16] has completed the result by considering the case of a ferrite with loss in first order approximation.

If on the other hand the radius of the ferrite rod approaches the radius of the guide, (21) holds with[5]

$$k_G = k_G^{(\epsilon_F)}\left\{1 - \frac{1}{s_{11}^2-1}(\epsilon_F - \epsilon)\left(\frac{k}{k_G^{(\epsilon_F)}}\right)^2\frac{a-\rho}{a}\right\}.$$

Finally the two complementary cases can be treated.

[16] P. I. B. Clarricoats, "Some properties of circular wave guides containing ferrites," *Proc. IEE*, vol. 104, B Suppl., pp. 286–295; 1957.

For the thin walled ferrite tube ($\rho \approx a$) adjoining the guide one gets

$$\frac{\Delta k_G^{\mp}}{k_G} = \frac{1}{s_{11}^2-1}(\mu_1 - 1)\frac{a-\rho}{\rho} \pm \kappa\left(\frac{a-\rho}{a}\right)^2$$

with

$$k_G = k_G^{(\epsilon)}\left\{1 - \frac{1}{s_{11}^2-1}(\epsilon - \epsilon_F)\left(\frac{k}{k_G^{(\epsilon)}}\right)^2\frac{a-\rho}{a}\right\}$$

and for the wave guide filled with ferrite and an axial dielectric rod ($\rho \ll a$)

$$\frac{\Delta k_G^{\mp}}{k_G} = \frac{1}{2}(\mu_1 - 1) \pm \frac{1}{s_{11}^2-1}\kappa - \frac{1}{4}\frac{s_{11}^2}{(s_{11}^2-1)J_1^2(s_{11})}\{(\mu_1-1) \pm \kappa\}\left(\frac{\rho}{a}\right)^2$$

with

$$k_G = k_G^{(\epsilon_F)}\left\{1 + \frac{1}{4}\frac{s_{11}^2}{(s_{11}^2-1)J_1^2(s_{11})} \cdot (\epsilon - \epsilon_F)\left(\frac{k}{k_G^{(\epsilon_F)}}\right)^2\left(\frac{\rho}{a}\right)^2\right\}.$$

Magnified and Squared VSWR Responses for Microwave Reflection Coefficient Measurements*

R. W. BEATTY†

***Summary*—In conventional microwave impedance measuring instruments, the measured ratio of maximum to minimum detector signal level is ideally equal to the voltage standing-wave ratio (VSWR) of the termination. In this paper, it is shown how radically different types of response are obtainable in which the observed ratio may approximately equal the square of the VSWR or may be magnified any desired amount. Theory is given enabling accurate measurements by interesting techniques. Accuracies of 0.1 per cent in VSWR to 2.0 have been achieved using magnified response techniques.**

Introduction

In most microwave impedance measuring instruments, such as the idealized slotted line, the resonance line, and rotary standing-wave indicators, the ratio of the maximum to the minimum amplitude of the output to the detector is ideally equal to the voltage standing-wave ratio (VSWR) of the termination subjected to measurement.

* Manuscript received by the PGMTT, February 20, 1959.
† Radio Standards Lab., Nat. Bur. of Standards, Boulder, Colo.

Other radically different types of response are obtainable. The two responses to be discussed in this paper have been called magnified and squared VSWR responses for reasons which will become apparent.

A simplified explanation will first be given, followed by a more complete mathematical description.

The differences among responses are shown in Fig. 1, three response curves calculated for the same termination.

Simplified Explanations

Squared VSWR Response

A simplified explanation can be given for one system yielding squared VSWR response. Other systems which have been devised apparently do not permit simplified explanations and will not be thoroughly analyzed. Enough theory will be given however, to permit their use as measurement systems.

The system shown in the diagram in Fig. 2 consists of a straight section of uniform lossless waveguide (which may be either coaxial line or rectangular waveguide, for example) with oppositely located coupling probes for generator and detector. A short circuit which may be adjusted in position terminates one end of the uniform waveguide section while the other end is terminated in the sliding load to be measured. In this system it is necessary to vary the phase of the load by sliding it inside the waveguide, but in other systems to be described, this is not always required.

Referring to the simplified model of Fig. 2(b),

$$E = e\frac{Z_p}{Z_G + Z_p} \approx i_G Z_p = i_G \frac{1}{\dfrac{1}{Z_s'} + \dfrac{1}{Z_L'}}\,. \qquad (1)$$

If the short circuit is located $\lambda_G/4$ from the probes, $Z'_s = \infty$, and

$$E \approx i_G Z_L' = i_G \frac{1 + \Gamma_L e^{-j2\beta l}}{1 - \Gamma_L e^{-j2\beta l}}\,. \qquad (2)$$

As l varies, $|E|$ goes through maxima and minima. The ratio σ_A of the maxima to minima is

$$\sigma_A = \left(\frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}\right)^2 = \sigma_L^2. \qquad (3)$$

Where σ_L is the VSWR of the load. The meaning of other symbols used above should become clear upon reference to Fig. 2(b).

Magnified Response

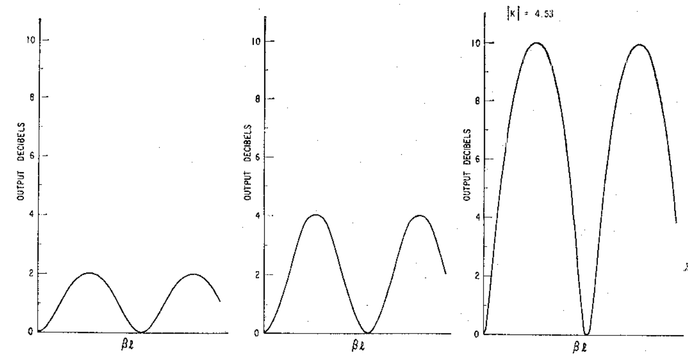
A system yielding magnified response is shown in Fig. 3. A directional coupler is connected to respond mainly to the wave reflected from a phasable or sliding termination whose VSWR is to be measured. For simplicity, it is assumed that the generator and detector do not produce reflections ($\Gamma_G = \Gamma_D = 0$), and that no reflections are produced in arm 2 by the directional coupler ($S_{22} = 0$).

The signal coupled to the detector has two components. One is fixed and exists because the directivity is not infinite. The other is from the load reflection and varies in phase as the load is slid inside the waveguide. As the relative phase of the two components vary, the magnitude of the resultant varies. If the components are of approximately equal magnitudes, the range of variation of the resultant may be large even though the reflection from the termination may be small.

Inspection of the diagram of Fig. 3 leads to the following equations describing the response.[1]

$$b_3 \approx b_G(S_{31} + S_{21}S_{32}\Gamma_L e^{-j2\beta l}) = b_G S_{31}(1 + K\Gamma_L e^{-j2\beta l}). \quad (4)$$

The response is of the same form as that of the idealized slotted line (see Fig. 1) excepting that Γ_L is multiplied by the factor K. Since $|K|$ may be very large (it approximately equals the directivity ratio of the directional coupler), one may consider Γ_L to be magnified by the factor $|K|$, leading to the term magnified response. It should be noted however that the response variation will increase as one increases $|K|$ up to the point where $|K\Gamma_L| = 1$, and then will decrease with further increase in $|K|$.



(a) (b) (c)

Fig. 1—Response curves of three measurement systems with termination having a VSWR of 1.26;

(a) idealized slotted line,

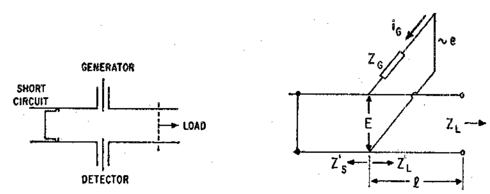
$$d = c[1 + K\Gamma_1 e^{-j2\beta l}]$$

(b) squared VSWR response,

$$d = c'\left[\frac{1 + \Gamma_L e^{-j2\beta l}}{1 - \Gamma_L e^{-j2\beta l}}\right],$$

(c) magnified response,

$$d = c''[1 + K\Gamma_L e^{-j2\beta l}].$$



(a) (b)

Fig. 2—Diagram and simplified model of one system yielding squared VSWR response; (a) diagram of system, (b) simplified model.

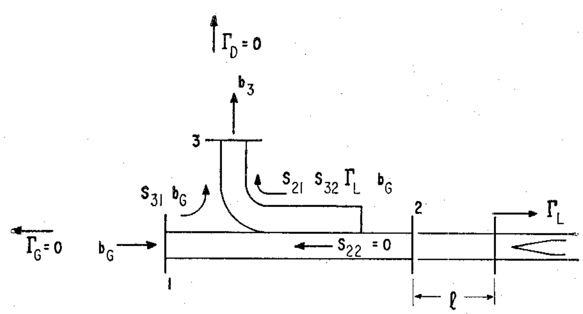


Fig. 3—Diagram of simplified system to illustrate magnified response.

ANALYSIS

Both magnified and squared VSWR responses may be analyzed by considering the generalized treatment[2]

[1] In this equation, b represents a wave amplitude, S a scattering coefficient of the directional coupler, and Γ_L the voltage reflection coefficient of the load.

[2] A. C. MacPherson and D. M. Kerns, "A new technique for the measurement of microwave standing-wave ratios," PROC. IRE, vol. 44, pp. 1024–1030, August, 1956.

of MacPherson and Kerns of a 3-arm junction measurement system for phasable loads. Such a junction is shown in Fig. 4. where it has been assumed for convenience that the necessary variation in phase is obtained by changing the length l of uniform, lossless waveguide.

Instead of using the gathering coefficients employed by MacPherson and Kerns, the solution[3] for b_3 is obtained in terms of the more familiar scattering coefficients and may be expressed as follows:

$$b_3 = C\frac{1 + K\Gamma_L e^{-j2\beta l}}{1 - \Gamma_{2i}\Gamma_L e^{-j2\beta l}} = CK\Gamma_L e^{-j2\beta l}\frac{1 + \dfrac{1}{K\Gamma_L}e^{j2\beta l}}{1 - \Gamma_{2i}\Gamma_L e^{-j2\beta l}} \qquad (5)$$

where

$$C = \frac{b_G S_{31}}{\begin{vmatrix} (1 - S_{11}\Gamma_G) & S_{13}\Gamma_D \\ S_{31}\Gamma_G & (1 - S_{33}\Gamma_D) \end{vmatrix}},$$

$$K = \frac{S_{21}S_{32}}{S_{31}} - S_{22},$$

and

$$\Gamma_{2i} = \frac{\begin{vmatrix} (1 - S_{11}\Gamma_G) & S_{12} & -S_{13}\Gamma_D \\ -S_{21}\Gamma_G & S_{22} & -S_{23}\Gamma_D \\ -S_{31}\Gamma_G & S_{32} & (1 - S_{33}\Gamma_D) \end{vmatrix}}{\begin{vmatrix} (1 - S_{11}\Gamma_G) & S_{13}\Gamma_D \\ S_{31}\Gamma_G & (1 - S_{33}\Gamma_D) \end{vmatrix}}.$$

In the above expressions, the component of the emergent wave amplitude supplied by the generator is $b_G = a_1 - b_1\Gamma_G$, where a_1 represents the amplitude of the wave incident on the junction in arm 1. Symbols of the form $S_{m,n}$ are the scattering coefficients of the junction, and Γ_G, Γ_D, and Γ_L are the voltage reflection coefficients of the generator, detector, and load, respectively, as indicated in Fig. 4.

The reflection coefficient Γ_{2i} is that which would be obtained "looking into" arm 2 if the generator was turned off and its impedance (as observed at T_1) was unchanged in so doing.

The variation in $|b_3|$ as we vary the phase (ψ_L) of Γ_L is defined to be the response of the systems represented by Fig. 4, and is determined by (5).

The properties of (5) will be examined in an effort to classify types of responses obtainable. It is evident that the parameter C affects only the level of the response, while the form of the response curve ($|b_3|$ vs ψ_L) is affected by the parameters K and Γ_{2i}.

We may consider the response for the conditions $\Gamma_{2i}=0$, $|K|=1$, the usual or normal type of response,

[3] G. E. Schafer and R. W. Beatty, "A method for measuring the directivity of directional couplers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 419–422; October, 1958.

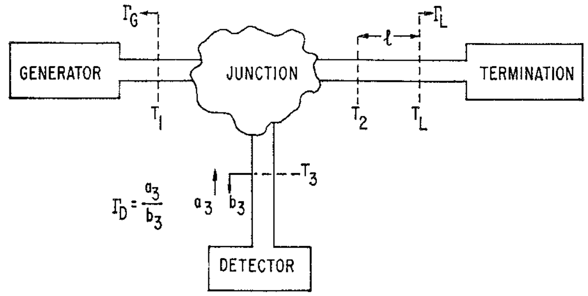


Fig. 4—Three-arm junction with phasable load.

since it leads to the form obtained in the case of an idealized slotted line.

The response form previously referred to as magnified response is obtained when $\Gamma_{2i}=0$, and $|K|$ is unrestricted. However values of $|K|$ may range from zero to infinity, and it is possible to have a magnification factor $|K|$ greater or less than unity. The case where $|K|$ is less than one is of dubious interest, but the possibility of $|K|$ greater than unity appears especially attractive for the measurement of small reflections.

If Γ_{2i} differs from zero, it will cause distortion in the response curve, which is considered undesirable. However, a distinctly different type of response may be obtained if $|\Gamma_{2i}| \approx 1$, and if the phases of $|K|$ and $|\Gamma_{2i}|$ are equal. This leads to squared VSWR response if $|K| \approx 1$. This type of response is not only curious, but may prove useful in some measurement applications.

Actually $|\Gamma_{2i}|$ is less than unity in actual (not lossless) systems, so that the ideal squared VSWR response may be closely approached with an actual system, but never quite reached.

A fourth type of response is obtained if the phases of K and Γ_{2i} are the same, $|\Gamma_{2i}| \approx 1$, and $|K|$ is unrestricted. The ratio of maximum to minimum detector signal level corresponding to (3) is

$$\sigma_A = \frac{1 + |K\Gamma_L|}{1 - |K\Gamma_L|}\sigma_L. \qquad (6)$$

It seems appropriate to call this a magnified squared VSWR response, and it may have applications in the measurement of large VSWR.

This completes the classification of responses, since conditions other than those mentioned may be regarded as causing distortions of the types described above.

Means of Obtaining Various Responses

Examples have already been given (Figs. 2 and 3) of junctions permitting magnified and squared VSWR responses. However other types of arrangements are possible and offer a variety of measurement systems, each with its possible advantages and disadvantages.

In order to closely approach squared VSWR response ($|\Gamma_{2i}| \approx 1$, $|K| \approx 1$) it becomes evident that the 3-arm junction should have low loss and low coupling to the load. (This may be shown from a consideration of the conditions imposed upon the scattering coefficients by losslessness.) These conditions are not sufficient however, as one may conclude after trying junctions which satisfy only these conditions. It is necessary for K and Γ_{2i} to have the same phase, and this is obtained by some tuning device, such as the adjustable short-circuit in Fig. 2. There may be some difficulty in obtaining the desired response in some cases, because it is not always possible to obtain the correct phase relationship, but the junction forms represented in Fig. 5(a) to (d) have all been found experimentally to permit a close approach to squared VSWR response by proper adjustment of the tuner. The arrangement of 5(e) should also permit squared VSWR response, but has not been constructed or tested.

Magnified Response occurs upon making $|K|$ greater than unity while $\Gamma_{2i}=0$. It evidently cannot be obtained with a lossless junction, for then $|K|=1$. If it is assumed that we can always make $\Gamma_G=0$, then Γ_{2i} would equal S_{22}, and this would vanish, so that $K=S_{21}S_{32}/S_{31}$. The directional coupler connected as shown in Fig. 3 evidently permits magnified response since $|S_{32}/S_{31}|$ is the directivity ratio and may be quite large while $|S_{21}|$ is usually between 0.7 and 1.0. The use of auxiliary tuners[4] with a directional coupler permits greater versatility since one may adjust the directivity ratio upwards or downwards with one tuner, then adjust the other tuner to make $\Gamma_{2i}=0$. These adjustments are independent only if made in the order described. Referring to Fig. 6, the tuner in arm 2 is adjusted first in order to obtain the desired value of $|K|$, then the tuner in arm 1 is adjusted to make $\Gamma_{2i}=0$.

Measurements Using Squared VSWR Response

Any of the junctions of Fig. 5 or their equivalents may be used if the unknown is phasable. This requirement is satisfied if the unknown termination slides inside the waveguide. In principle, a phase shifter or line stretcher may also be used to provide the phase variation, but in practice, they are less than perfect, leading to additional errors in measurement. If the unknown termination does not slide within the waveguide, the arrangements of Fig. 5(d) and (e) may be used. Either flexible cables must be used to couple the generator and detector to the moving junction, or the generator and detector may be arranged to move with the junction. If the termination is not too large, it and the waveguide section could be moved, keeping everything else fixed. The arrangement shown for rectangular waveguide is not readily adaptable to operation with coaxial lines. However the arrangement of Fig. 5(e) should be satisfactory for operation with coaxial line.

The correct adjustment of the tuner is made with $\Gamma_L=0$ and corresponds to maximum detector output for Figs. 2 and 5(a) and to minimum detector output for the others shown. When the correct adjustment has

[4] G. F. Engen and R. W. Beatty, "Microwave Reflectometer Techniques," this issue, p. 351.

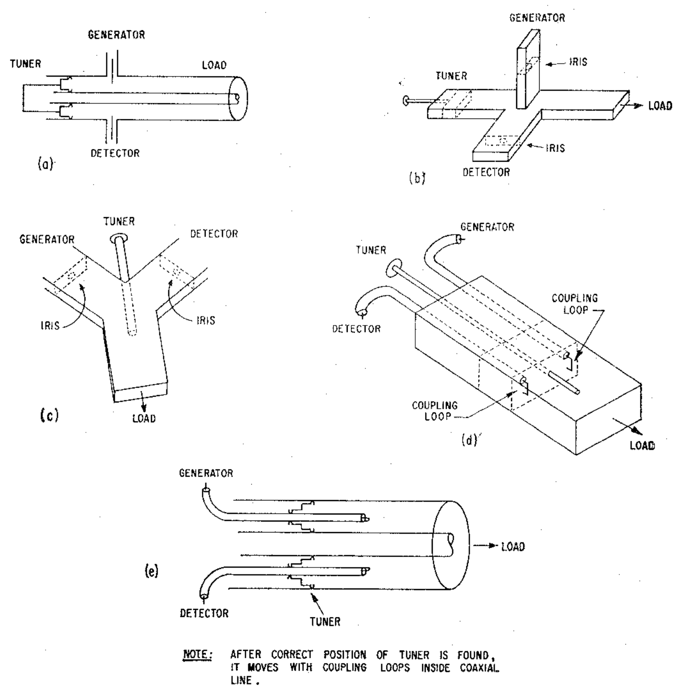


Fig. 5—Schematic drawings of junctions permitting squared VSWR response. (a), (b), (c), (d), (e).

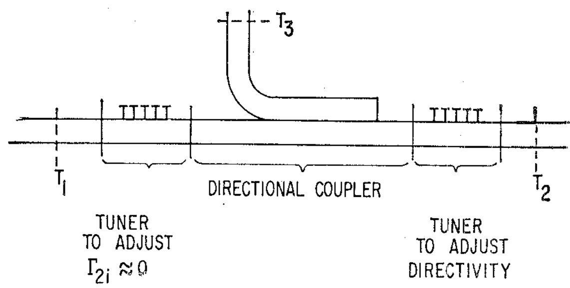


Fig. 6—Directional coupler with auxiliary tuners.

been made, the response curves will be symmetrical about the maxima and minima.

A correction may be made for deviations from the ideal conditions $|K|\approx 1$, $|\Gamma_{2i}|\approx 1$, using the methods indicated[2] by MacPherson and Kerns. Instead of analyzing the response curve to obtain the parameter y required for the correction, a shorter method is as follows. Only the real part (g) of y is needed for a correction to the VSWR. It can be shown that to a good approximation (to the first order in g and b),

$$\sigma_I = \frac{1+|K\Gamma_L|}{1-|K\Gamma_L|} = \sqrt{\sigma_A} + \tfrac{1}{2}(g+1)(\sigma_A - 1), \quad (7)$$

where

$$(g+1) \approx \frac{|b_3|\,(\Gamma_L = 0)}{|b_3|_{\max}(|\Gamma_L| = 1)}.$$

It is still necessary to determine $|K|$ in order to obtain $|\Gamma_L|$ or σ_L. This may be done by measuring σ_I when a termination of known $|\Gamma_L|$ is connected.

Measurements Using Magnified Response

The arrangement of Fig. 6 may be used to measure the voltage reflection coefficient Γ_U of an unknown termination. Two basic methods[4,5] will be outlined.

In the first method, the auxiliary tuners are adjusted for the conditions $\Gamma_{2i}=0$ and $K=\infty$. Inspection of (5) shows that $|b_3|$, the magnitude of the detector arm wave amplitude will then be proportional to $|\Gamma_L|$. One then measures the ratio r of the $|b_3|$ values obtained when the load is first unknown (Γ_U), then a standard of known reflection coefficient magnitude $|\Gamma_S|$. Then

$$|\Gamma_U| = r|\Gamma_S|. \quad (8)$$

The adjustments of the tuners preceding the measurement is as follows.[4] One adjusts the tuner in arm 2 until no variation is observed in $|b_3|$ as one slides a termination of low reflection inside the output waveguide. Then the tuner in arm 1 is adjusted until no variation in $|b_3|$ is observed as one slides a termination of high reflection inside the output waveguide. If necessary, the above operations are repeated in sequence until no variation in $|b_3|$ is observed as either termination is slid.

In the second method, the tuner in arm 2 is adjusted (with the unknown connected to arm 2) until the detector output is zero. Then $K\Gamma_U=-1$. The tuner in arm 1 is then adjusted until $\Gamma_{2i}=0$. A reflection standard of known $|\Gamma_S|$ is then connected to arm 2, and the phase of Γ_S is varied. Substitution of the above conditions into (5) leads to

$$\sigma_A = \frac{|b_3|_{\max}}{|b_3|_{\min}} = \frac{|\Gamma_U| + |\Gamma_S|}{\big|\,|\Gamma_U| - |\Gamma_S|\,\big|}. \quad (9)$$

In the event that $|\Gamma_S| \approx 1$, (approximately true for a sliding short-circuit), then $\sigma_A = \sigma_U$.

Note that it is unnecessary to vary the phase of Γ_U, the reflection coefficient of the unknown termination in either method. In the second method one needs to vary the phase of Γ_S, but this is easily done if a sliding short-circuit is used.

Alternatively a fixed reflection standard may be used if a suitable line stretcher is incorporated into arm 2 of the measuring instrument.

[5] R. W. Beatty and D. M. Kerns, "Recently developed microwave impedance standards and methods of measurement," IRE Trans. on Instrumentation, vol. I-7, pp. 319–321; December, 1958.

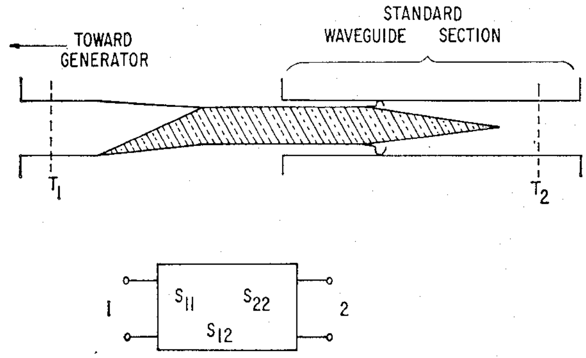


Fig. 7—Line stretcher and scattering coefficient representation.

Because of the special condition $\Gamma_{2i}=0$, the line stretcher need not be of the constant impedance type, since reflections that it may introduce may be cancelled by reflections from the tuner in arm 1. Also, the reference plane for arm 2 may be located in the uniform waveguide section of the line stretcher between the source of its reflections and the load. As shown in Fig. 7, the reference plane T_2 remains fixed although the output waveguide and load move. The line stretcher must be stable however, so that the parameters S_{11}, S_{12}, and S_{22} with respect to the reference planes T_1 and T_2 do not vary as it operates.

With the addition of the line stretcher, the second method described above may be called a magnified difference method, since the smaller the difference between $|\Gamma_S|$ and $|\Gamma_U|$, the greater the variation in $|b_3|$ as the phase is changed (9).

Evaluation

It is too early to make a conclusive evaluation of the worth of the responses described above and their applications in measurement systems. However, the techniques employing magnified response give promise of increased accuracy in the measurement of low and intermediate VSWR. Accuracies of approximately 0.1 per cent in VSWR to 2.0 have been achieved, and perhaps an order of magnitude better than that is possible.

Acknowledgment

Dr. D. M. Kerns made many helpful suggestions, including the junction of Fig. 5(b), and W. J. Anson and E. Niesen made measurements to verify the techniques described above.